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Электронный научный журнал «Студенческий форум» отражает результаты научных исследований, проведенных представителями различных школ и направлений современной науки.

Данное издание будет полезно магистрам, студентам, исследователям и всем интересующимся актуальным состоянием и тенденциями развития современной науки.

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Оглавление

PAPERS IN ENGLISH

RUBRIC

«LIFE SAFETY»

AIR POLLUTION IN KAZAKHSTAN

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What could be done to reduce smog in Kazakhstan?

The problem of air pollution in Kazakhstan is becoming more and more urgent. According to a World Bank study released in December 2021, Kazakhstan loses 6,000 to 9,360 individuals prematurely each year because of poor air quality (*Air Quality and Decarbonization: How to Balance Two Important Issues*, 2022). Thus, the main goal of the study is identifying and propose effective measures to reduce the risks caused by smog and improve air quality in Kazakhstan. The study assesses the current state of air pollution in the country and looks for practical recommendations to reduce smog level and associated environmental and health risks. In addition, with the help of primary sources, especially by conducting a survey that collects information about the experiences of residents and analyzes them from the perspective of residents.

In a result, the survey and study found that reducing potential pollution and improving air quality in Kazakhstan requires a multifaceted approach, including stricter vehicle emission regulations, industrial technology upgrades, promotion of renewable energy sources, and improved energy efficiency (*About the Concept for the Transition of the Republic of Kazakhstan Towards a Green Economy*, 2013). The current actions of the commitment to carbon Neutrality and initiatives to expand the renewable energy market are positive steps towards reducing air pollution and achieving sustainable development in the country (*First Ever Local Auction to Boost Kazakhstan's Renewable Energy Market*, 2020), but unfortunately not all citizens are informed about these actions of the state. "Decarbonizing the energy sector can bring two benefits: reduce carbon emissions and contribute to mitigating climate change; and by switching to cleaner fuels, reduce emissions of pollutants into the atmosphere from fuel combustion," notes Andrey Mikhnev, World Bank Resident Representative in Kazakhstan (World Bank in Kazakhstan, 2023). Residents, in turn, fully understand the harm and causes of smog, further public support will help citizens to individually take action in the fight against smog.

How to reduce the industry's impact on pollution

Secondly, outdated industry technologies have a strong negative impact on air quality. In Kazakhstan, where almost 70 percent of electricity is generated from coal, PM2.5 levels are 4.4 times higher than the WHO recommended levels. The main sources of air pollution in cities are stationary types of thermal power plants, boiler houses, vehicle emissions, construction sites, cement plants and asphalt concrete plants. Andrey Mikhnev, World Bank Representative in Kazakhstan notes: "Today, the energy sector accounts for approximately 80% of carbon emissions in Kazakhstan. Therefore, decarbonizing the energy sector can have two benefits: it can reduce carbon emissions, which helps mitigate climate change, and it can reduce air pollution from fuel combustion through a switch to cleaner fuels” (World Bank in Kazakhstan, 2023). Moreover, energy efficiency is a measure of a country's industrial sector's potential to impact air quality. indicates the use of outdated technologies that tend to emit more air pollutants (United Nations Environment Programme, 2016). By 2030, alternative solar energy could provide up to 15% of Kazakhstan's energy needs, making it an important alternative energy source for the rural population within a decade. Currently, 2.3 percent of the total energy production in the country comes from renewable sources (First-ever local auction for the development of the renewable energy market in Kazakhstan, 2020).

Conclusion

To summarize, technological advances have now brought us to one of the major problems of air pollution. Dust is classified as hazard class three, but there may be toxic chemical elements in general urban dust, which unfortunately hygienists rarely take into account. Therefore, to protect citizens, the causes must be fully investigated, and measures taken to clean the air as much as possible. It is not only eco-activists who are involved, but also the state. For example, to counteract the adverse effects of accelerated economic growth, the Government of Kazakhstan adopted a new Environmental Code on 9 January 2007, which regulates all aspects of processes affecting the environment, in particular emissions of gases and other pollutants into the atmosphere, contains general rules used to control and industrial emissions in Kazakhstan. Despite this, the country's emission standards are significantly higher than in Europe (*About the Concept for the Transition of the Republic of Kazakhstan Towards a Green Economy*, 2013).

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RUBRIC

«TECHNICAL SCIENCES»

RECOMMENDATION SYSTEM METHODS FOR ONLINE SUPERMARKETS USING AI

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МЕТОДЫ РЕКОМЕНДАТЕЛЬНОЙ СИСТЕМЫ ДЛЯ ОНЛАЙН СУПЕРМАРКЕТОВ ИСПОЛЬЗУЯ ИИ

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Аннотация. Ценная информация может быть использована продуктовыми ритейлерами для улучшения своих маркетинговых стратегий и операционной деятельности. В этой статье автор предлагает изучить алгоритмы машинного обучения для разработки персонализированной системы рекомендаций по спискам покупок. Предлагаемая система предназначена для рассмотрения различных типов алгоритмов, которые могут быть интегрированы с различными интернет-супермаркетами.

Abstract. Valuable information can be used by grocery retailers to improve their marketing strategies and operational activities. In this article, the author suggests studying machine learning algorithms to develop a personalized system of recommendations for shopping lists. The proposed system is designed to consider various types of algorithms that can be integrated with various online supermarkets.

Ключевые слова: Искусственный интеллект, матрица факторизации, рекомендательная система, машинное обучение.

Keywords: Artificial Intelligence, factorization matrix, recommendation system, machine learning.

Recommendation systems powered by artificial intelligence (AI) have emerged as a solution to this problem. Recommendation systems use machine learning algorithms to analyze customer data, such as purchase history and search queries, to provide personalized recommendations for products that the customer may be interested in. In the context of online supermarkets, these systems can increase customer satisfaction by helping them find the products they need more efficiently, while also increasing sales for the supermarket.

The recommender system is designed to be expandable and customizable, allowing the possibility of training the model with data from different retailers. Both the quality of predictions and the performance of the solution are crucial factors in achieving the desired outcome, and thus, they are thoroughly measured and analyzed. To evaluate the quality of the recommender system, two non-machine learning approaches provided by the organization are utilized for comparison. Additionally, the performance is evaluated under a simulated workload to assess its efficiency [1].

Recommendation systems can be evaluated using metrics such as accuracy, coverage, and diversity. Accuracy measures how well the system predicts the preferences of the user, while coverage measures how many items in the catalog the system can recommend. Diversity measures how much variety there is in the recommended items. These metrics can help developers optimize the performance of the recommendation system and improve the customer experience.

Reliability in the context of a system refers to its behavior and encompasses aspects such as availability, precision, and failure recovery. In the scope of this project, specific reliability requirements have been identified, which are outlined below:

1. The system should maintain its availability even after experiencing an error in a previous operation. This means that if an error occurs, the system should handle it appropriately and continue functioning without causing a complete system failure or disruption.

2. The system should effectively handle errors that may occur within internal operations or requests, ensuring that these errors are contained and do not propagate to other components or adversely affect the overall system functionality. It is essential to implement mechanisms and strategies to isolate and manage errors internally, minimizing their impact on the system's stability and performance.

By adhering to these reliability requirements, the recommender system can maintain its availability, continue providing accurate recommendations, and ensure that errors are contained within the affected components without causing system-wide failures. This contributes to a more robust and reliable system overall [2].

The sections below present some of the most typical classifications for recommender systems, according to the literature:

1. Content-Based Recommendations.
2. Collaborative Recommendations.
3. Demographic-Based Recommendations.
4. Interactive Recommendations.
5. Context-Based Recommendations.

Content-based recommender systems, also called cognitive filter recommender systems, are well-known for their simplicity. Collaborative recommender systems, also referred to as "social filter" or "collaborative filtering" recommender systems, make recommendations by analyzing users' behavior. Demographic-based recommender systems are hybrid approaches that use user-user relationships to generate predictions. Different solutions may collect this information in various ways, but they aim to gather data that enables the system to calculate user profiles.

However, generating knowledge from contextual information can be challenging, making this approach complex to implement, despite its promising results and academic interest. Picture 1 shows the most basic types of machine learning (ML).

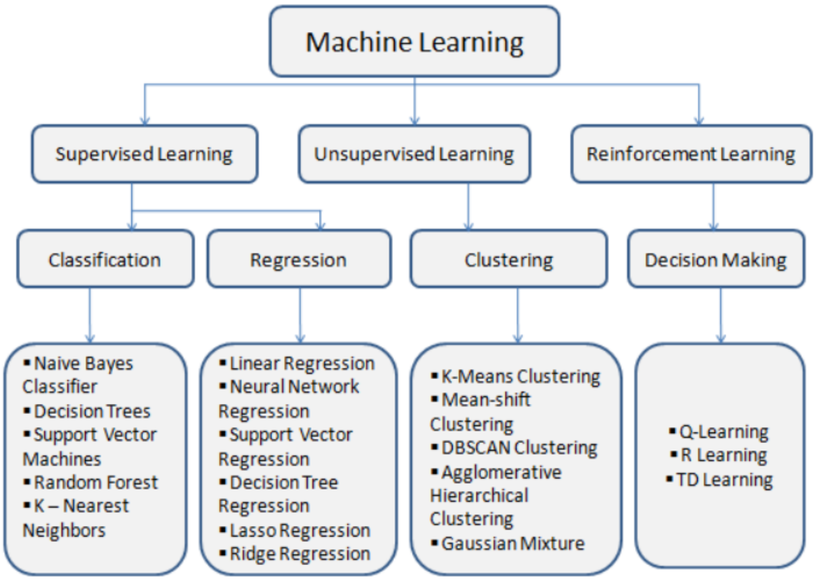

Figure 1. Types of machine learning

The primary objective of AI is to simulate human-like intelligence in order to solve complex problems, while ML aims to learn from collected data to enhance the machine's performance for a specific task. Thus, ML can be seen as a system that learns new information from the data it gathers [3]. When it comes to decision-making, AI strives to obtain the most suitable solution, while ML focuses on finding a single answer, regardless of whether it is optimal or not. AI leads to the development of intelligence and wisdom, whereas ML leads to knowledge acquisition. It is worth noting that while AI works towards making decisions and arriving at optimal solutions, ML is limited to considering a single answer.

Matrix Factorization – it decomposes the utility matrix into two sub matrices. During prediction, we multiply the two sub-matrices to reconstruct the predicted utility matrix. The utility matrix is factorized such that the loss between the multiplication of these two and the true utility matrix is minimized. One commonly used loss function is mean-squared error. Example on Picture 2.

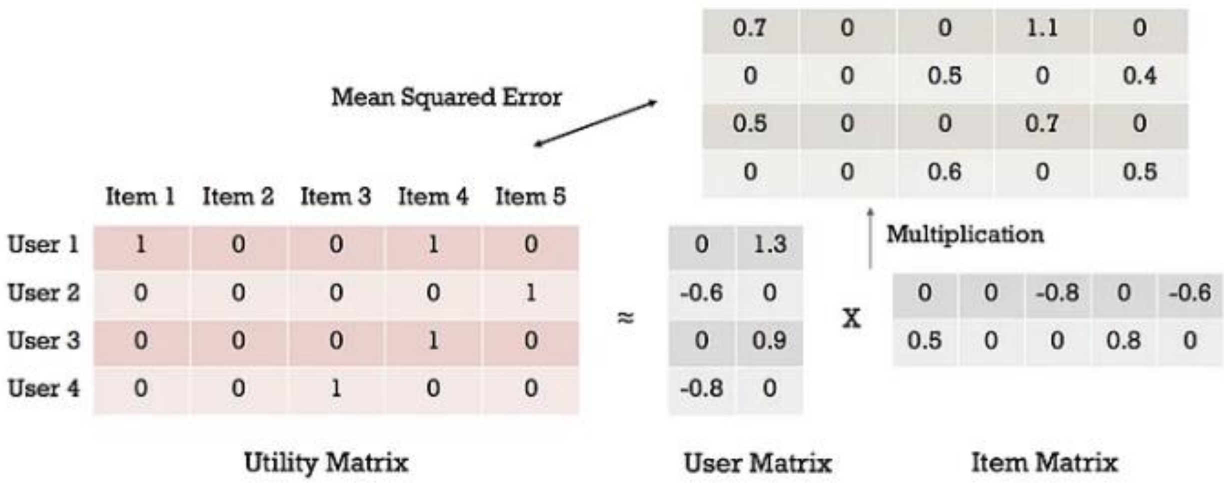

Figure 2. utility matrix

Matrix Factorization is a popular technique used in recommendation systems to model and predict user-item interactions. It aims to factorize a user-item interaction matrix into two lower-dimensional matrices: one representing user and the other representing items.

In summary, Matrix Factorization is a technique used in recommendation systems to decompose the user-item interaction matrix into lower-dimensional matrices and extract latent factors. It enables personalized recommendations by predicting missing entries in the matrix [4]. While it has advantages in handling sparsity and scalability, it may have limitations in capturing complex relationships and incorporating contextual information. Hybrid approaches can be used to overcome these limitations and enhance recommendation accuracy.

Accuracy is used as an evaluation metric in recommendation systems to assess the effectiveness and performance of the system's recommendations. It measures how accurately the recommendations align with the preferences or interests of the users.

In neural networks, accuracy is a measure of how well the model is able to correctly classify examples from the test dataset. It is calculated as the ratio of the number of correctly classified examples to the total number of examples in the test dataset. In other words, accuracy measures the percentage of correct predictions made by the model on the test dataset.

Let's check the results of our accuracy in the diagram on Picture 3.

```
plt.plot(hist.history['accuracy'])
plt.plot(hist.history['val_accuracy'])
plt.title('model accuracy')
plt.ylabel('accuracy')
plt.xlabel('epoch')
plt.legend(['train', 'val'], loc='upper left')
plt.show()
```

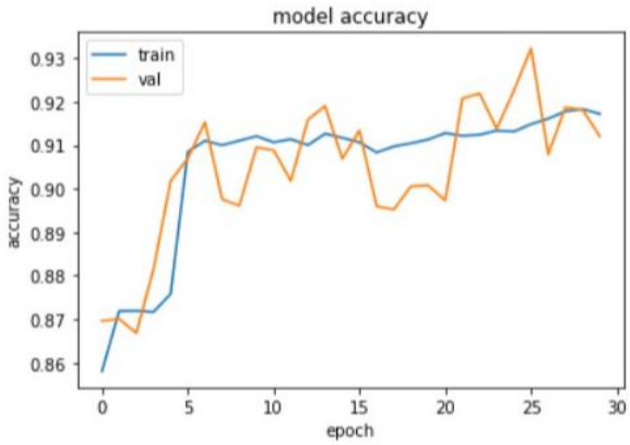

Figure 3. Accuracy for 30 epochs

Based on this research, the author identified gradient boosted tree algorithms, neural networks, and SVMs as the most promising approaches for developing a machine learning model for grocery retail [5]. These techniques were found to deliver the best results and show greater consistency across different domains. The passage also notes that there are various technologies available for developing machine learning algorithms, and TensorFlow was chosen as the framework for this dissertation due to its popularity and strong community support. Additionally, TensorFlow provides support for both CPU and GPU, which leads to faster results when working with models.

Developing a recommender system for grocery retail is a challenging task due to the unique characteristics of the industry. In order to create an effective model, it was crucial to extract as much knowledge as possible from the dataset used in the case study. This knowledge was translated into features, which played a critical role in helping the gradient boosted trees model learn patterns that are specific to different customers, products, and their relationships.

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RUBRIC

«PHYSICAL AND MATHEMATICAL SCIENCES»

SUBSPACES OF VECTOR SPACES

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Abstract. This article is designed to study various existing subspaces of linear spaces. The conclusions concluded in the practical part allow us to consider the ways of solving and applying them in practice and compare these subspaces with each other

Keywords: Vector space, subspace, linear space.

The set L of vectors of a linear space X is called a subspace if it follows from the fact that the vectors x, y belong to L that the vector ax + βy for any complex numbers α, β also belongs to the set L. [1]

1) Zero space {o}. In this case, V = {o}. Any field can be considered as F. The operations are defined in a trivial way: o+ o =o, αo=o. It is easy to verify that the specified set with such operations is a linear space over the field F." [2]

«2) Geometric spaces V_1 , V_2 , V_3 . The elements of this space are geometric vectors, i.e. directed segments, in space. A geometric vector whose beginning is at point A and the end is at point B is denoted by $AB^{\rightarrow}$. Two vectors are considered equal if they are co-directed and have the same length. In view of this, it is convenient to assume that all vectors are fixed at one point O, called the pole or the origin. This consideration is also convenient because each vector is associated with a certain point of space – its end, and, conversely, a single vector is associated with each point of space, called the radius vector of the point, the beginning of which is fixed at the pole, and the end points to this point. Vectors are added according to the parallelogram rule: the sum of two radius vectors is the diagonal of a parallelogram built on these vectors. Vectors can be multiplied by real numbers. The product of the radius vector by the number α means a vector whose length is equal to the length of the original vector multiplied by |α|, and the direction coincides with the direction of the original vector if α > 0, and is replaced by the opposite if α < 0. Thus, V_3 is a linear space over the field R. Analogous sets of vectors on a plane and on a straight line are denoted by V_2, V_1 accordingly. These aggregates are also linear spaces over the field R.» [2]

«3) Arithmetic space F_n. The elements of this space are columns of height n, composed of numbers. Two columns

$$a = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_n \end{pmatrix}, b = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \cdots \\ \beta_n \end{pmatrix}.$$

we call them equal if $\alpha_j = \beta_j$ $(j = 1,2,\dots,n)$. Numbers α_j и β_j are called components of vectors a and b accordingly. The operations of adding columns and multiplying them by numbers from F are defined according to the following rules:

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \dots\\ \alpha_n\end{pmatrix}+\begin{pmatrix}\beta_1\\ \beta_2\\ \dots\\ \beta_n\end{pmatrix}=\begin{pmatrix}\alpha_1 & + & \beta_1\\ \alpha_2 & + & \beta_2\\ \dots & + & \dots\\ \alpha_n & + & \beta_n\end{pmatrix}, \alpha\begin{pmatrix}\alpha_1\\ \alpha_2\\ \dots\\ \alpha_n\end{pmatrix}=\begin{pmatrix}\alpha\alpha_1\\ \alpha\alpha_2\\ \dots\\ \alpha\alpha_n\end{pmatrix}$$

It is also easy to check that F_n forms a linear space over the field F, called an arithmetic n-dimensional space. In particular, the zero vector in space F_n is

$$0 = \begin{pmatrix}0\\ 0\\ \dots\\ 0\end{pmatrix}.$$» [1]

«4) The space of polynomials F[x]. The elements of this space are polynomials with coefficients from the field F. It is easy to see that with respect to the usual operations of addition and multiplication of polynomials by numbers from F, the set V = F[x] forms a linear space over the field F. In particular, the zero vector in this space is the zero polynomial.» [2]

«5) Matrix space $F^{m\times n}$. The elements of this space are matrices of given dimensions m × n with elements from the field A. It is easy to see that with respect to the operation of matrix addition and multiplication of matrices by numbers from A, the set $V = F^{m\times n}$ forms a linear space over the field F. In particular, the null vector in this space is the null m × n matrix.» [2]

We have considered some of the subspaces of linear space. Each of them is used in mathematics to achieve certain goals.

For example, take the vector space V, there is also a subspace: W2, W2.

Sometimes the intersection of two subspaces contains only the zero vector. If a set containing only a null vector were not considered a vector space, the statement in the first paragraph would be false. So one of the reasons why a vector space containing only a null vector is useful when answering questions about linear algebra is that it saves us from special cases that we have to pay special attention to. We would rather write "the intersection of W1 and W1 is a subspace" than write "the intersection of W2 and W2 is either a subspace or a set containing only a null vector".

Geometric spaces are also necessary, for example, in a metric space we can determine the distance between two points. Or the "event space", which plays an important role in the geometric interpretation of the theory of relativity. If it is necessary to work with numbers, then there is an arithmetic space for this. It defines the operations of addition and multiplication of numbers. It is advisable to work in the space of polynomials if the operations of addition and multiplication according to the conditions of the problem are defined over a set of polynomials, and if there is a need to perform actions with matrices, then we will use the matrix space.

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IRREDUCIBILITY OF POLYNOMIALS OVER VARIOUS NUMBER FIELDS

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Abstract. The article considers irreducibility of polynomials over various number fields. Presently, in algebra and number theory, it is possible to compose many polynomials of different degrees, but each of them may have roots, so it can be decomposed into prime factors, or it may not be.

Keywords: mathematics, algebra, polynomials, number fields.

The purpose of this research is to study the concept and properties of irreducible polynomials in solving problems of algebra and number theory. The task of the research is to analyze the concepts of “polynomial” and “number field”, define the concepts of polynomial reducibility and irreducibility, identify signs of irreducibility, give problems to prove the irreducibility of polynomials.

Let us consider the basic concept of polynomials and number fields, and study their properties in depth, as well as solve problems of finding roots of a polynomial with rational coefficients.

1. Concepts of Polynomials and Their Properties in Algebra.

Let y=f(x) be a rational integral function. Its expression through X may have a different appearance. For example,

$$y = (x-2)^2(x+1)^2 = (x^2-4x+4)(x+1)^2 = (x-2)^2(x^2-2x+1)$$

It is possible to continue these expressions, but they will all define the same function. The question arises as to how this rational integral function can be represented in a “standard”, “canonical form”.

Definition. A record of rational integral function is considered canonical if it has no brackets and similar terms, and the summands in it are arranged in a descending order of exponents of X.

The reduction to the canonical form is done as follows: all brackets are expanded using the distributive law

$$a(b+c) = ab + ac$$

After expanding the brackets, all the products of degrees of the variable are replaced and similar summands are given and arranged in a descending order of exponents. The result is an expression of the form:

$$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n, a_0 \neq 0$$

Such an expression is called a polynomial of X, and n is the degree of this polynomial.

Numbers $a_0; a_1; \dots; a_n$ refer to coefficients of polynomial f(x). In particular, a_0 refers to a higher coefficient, a_n refers to the constant term.

2. Types of Number Fields and Their Properties in Algebra.

In abstract algebra, a field is a set for elements of which the operations of addition, taking the opposite value, multiplication and division (except for division by 0) are defined, wherein the properties of these operations are close to the properties of ordinary number operations. The simplest field is the field of rational numbers (fractions). Although the names of the field operations are taken from arithmetic, it should be kept in mind that the field elements are not necessarily numbers, and the definitions of the operations may be far from arithmetic. **An algebraic number field, field**

of algebraic numbers (or simply – **a number field**) is a finite (and, therefore, algebraic) extension of the field of rational numbers Q.

Thus, a number field is a field containing Q and is a finite-dimensional vector space over it. Number fields and, more general, algebraic extensions of the field of rational numbers are the main object of the study of algebraic number theory.

Example 1. Number fields Q, R, C are the main examples of fields for us.

Example 2. For each prime number p, there is a number field of residues Zp from p elements.

3. Irreducible and Reducible Polynomials

Definition 1: p(x) over P is irreducible if its divisors are divisors of form c and p(x) and it has no other divisors.

Definition 2: $f(x)$ over P is reducible if, in addition to divisors c and c*f(x), this polynomial has other divisors φ(x), where degree of φ(x) is any natural number.

The same polynomial over one field may be irreducible, and over another – reducible.

$f(x)$ of degree $n \geq 1$ is reducible over P if there are polynomials $\varphi(x), g(x)$ over P, that there is an equality

$$f(x) = \varphi(x) * g(x) \text{ at same degrees } n \geq 1$$

$p(x)\ of$ degree $n \geq 1$ is irreducible over P if in any of its representations of the form

$$p(x) = \varphi(x) * g(x)$$

one of the polynomials has 0 degree, the other – n.

4. Problems for Proving Irreducibility of Polynomials

Example 1. Polynomial $f(x)=x^2 + 1 = (x - i)(x + i)$ is reducible over field C and irreducible over fields Q and R.

Polynomial $h(x) = x^{2\,-}5x + 6 = (x - 3)(x - 2)$ is reducible over Q, R and C.

Example 2. Over which of the fields N, Q, R or C the polynomials are irreducible:

a) $\mathrm{f}\,(\mathrm{x}) = \mathrm{x}^2 - 10\mathrm{x} + 21$;

b) $\mathrm{h}(\mathrm{x}) = 3\mathrm{x}^2 + \mathrm{x} + 3$;

To solve these problems, let us find the roots of the quadratic equation by the formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $\mathrm{f}\,(\mathrm{x}) = \mathrm{x}^2 - 10\mathrm{x} + 21$ we have

$$x_{1,2} = \frac{10 \pm \sqrt{100 - 4 * 21 * 1}}{2 * 1} = \frac{10 \pm 4}{2} => \ x_1 = 7, x_2 = 3$$

Then, $\mathrm{f}\,(\mathrm{x}) = \mathrm{x}^2 - 10\mathrm{x} + 2 = (x - 7)(x - 3)$. This polynomial is reducible over all fields N, Q, R or C.

The second polynomial $\mathrm{h}(\mathrm{x}) = 3\mathrm{x}^2 + \mathrm{x} + 3$ has no real roots, but has imaginary roots

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 * 3 * 3}}{2 * 3} = \frac{-1 \pm \sqrt{37} * i}{6}$$

It means that it is irreducible over N, Q, R.

Conclusion. Polynomial $P(\propto)= a_0 + a_1 \propto + \dots + a_n \propto^n = 0$ of degree n by x is irreducible if it cannot be represented in a form of product $f(x) = \varphi(x) \cdot g(x)$ of polynomials $\varphi(x)$ and $g(x)$ with integral coefficients, but of degrees less than n. If similar is possible, then P(x) is reducible. Such different signs of irreducibility have been studied and formulated. Examples of problems for proving the irreducibility of polynomials in a number field are given. The application of irreducible polynomials in the factorization of set C is considered.

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