SOLVING ALGEBRAIC EQUATIONS IN RADICALS (BACKGROUND)

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Abstract. The problem of solvability of algebraic equations in radicals is considered in the article, and important steps in solving this problem are studied. In all formulas for obtaining the roots of algebraic equations, there are operations of addition, multiplication, subtraction, division, and extraction of roots of various degrees over the coefficients of the equation. In this connection, the concept of solving an equation in radicals appears.

Keywords: algebraic equations; radicals; exponent; factor; cubic equation; abelian equations.

Method of cubic equations in Tartaglia-Cardano radicals. Girolamo Cardano (1501-1576) was one of the pre-eminent and versatile scientists of the Renaissance. Mathematical works of Cardano – «Practice of general arithmetic and simple measurements», «Great Art or on the Rules of Algebra» and many others are collected in the fourth volume of the Lyon edition of the works of the Milanese. He wrote about almost everything that Renaissance mathematics knew, alternating new, own results with those that had already been obtained by other authors.

The main algebraic problem that occupied Cardano was to find ways to solve equations of the third and fourth degree. In accordance with the mathematical traditions of his time, he considered only equations with positive coefficients, therefore, for example, equation , where or , he divided into three separate cases: , , $=qx$ [2].

The first attempts to solve the cubic equation are already found in the "Practice of General Arithmetic and Simple Measurements". It is true that Cardano only succeeded in dealing with equations of a certain kind, but the methods he used are noteworthy, since he subsequently applied them successfully in The Great Art. He noticed that a cubic equation can sometimes be solved by adding the same expression to both sides so that a common divisor is formed that can be reduced. In this case, the solution of the cubic equation was reduced to the solution of the quadratic one.

This, in fact, ends the contribution of this scientist in solving the cubic equation, which for a long time was unfairly called the "Cardano formula". The fact is that the method for solving equation (4) was discovered by another Italian scientist Niccolò Tartaglia (1499–1557) on February 12, 1535 in preparation for a mathematical duel with a certain Fiore.

The question of who owns the discovery of a general method for solving cubic equations remains controversial, there are several different opinions. History turned out to be unfair to Tartaglia - the way to solve a cubic equation has long been known in mathematics under the name "Cardan's formulas".

Unsolvability in radicals of algebraic equations of degree greater than four. Mathematicians have long tried to find a method for solving the general equation of the 5th degree in radicals. The works of P. Ruffini and N.Kh. Abel were able to answer the question whether there are general formulas for solving equations of the fifth degree and higher. Their theorem is: the general algebraic equation of degree is unsolvable in radicals. However, this does not mean that it is impossible to solve some particular types of equations of higher degrees in radicals. In 1824, Niels Henrik Abel found such a solution for a wide class of equations of arbitrarily high degree - the so-called Abelian equations. The Abel-Ruffini theorem does not even exclude the fact that the roots of each particular algebraic equation can be written in terms of its coefficients, using the signs of arithmetic operations and radicals. In particular, that any algebraic number, the root of an equation with integer coefficients, can be expressed in radicals in terms of rational numbers. But such an expression does not always exist [1].

The next, final, step in the study of the solvability problem was made by E. Galois. Considering numerical equations, E. Galois established the concept of their group, that is, the set of permutations between their roots that do not violate the rational relationships between them. This group determines for each equation the algebraic structure of its roots. In particular, an equation is solvable in radicals if and only if its group is one of the so-called solvable groups. Therefore, the question of the solvability of each given equation in radicals can be solved by a finite number of actions [3].

The method for obtaining a solution to an equation of the fourth degree was published in 1545, along with the solution of a cubic equation, in D. Cardano's book "Great Art". In all formulas for obtaining the roots of algebraic equations, there are operations of addition, multiplication, subtraction, division, and extraction of roots of various degrees over the coefficients of the equation. In this connection, the concept of solving an equation in radicals appears.

The paper studies the history of obtaining general formulas in radicals for solving equations of degree not exceeding four. The problem solved by Abel and Galois on solving equations of degree greater than or equal to five in radicals is also briefly described.

The questions investigated in this work are important for the future teacher of mathematics, as they are connected with an important content line of the school course, with the problem of solving algebraic equations.

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