

## CUBIC EQUATIONS

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Consider the general concept of n-degree polynomials according to the textbook by E.B. Vinberg [1, p. 5].

Definition [1]: a polynomial of n-degree is a function of the form:

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n, \quad (1)$$

where  $a_0, a_1, a_2, \dots, a_n$  - are real numbers (some of them may be zero). The numbers  $a_n$  in equation (1) are called the coefficients of the function  $f(x)$  at  $x^n$ . A polynomial whose coefficients are all equal to zero is called zero.

Cubic equations are polynomials of the third degree, having the form:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3, \quad (2)$$

where  $a_0, a_1, a_2, a_3$  - are real numbers, with  $a_3 \neq 0$ .

Equally important is the concept of a reciprocal equation. The reciprocal equation is an equation of the form:

$$f(x) = a_2x^3 + a_1x^2 + a_1x + a_2 = 0,$$

where  $a_1, a_2 \neq 0$  [1].

Consider properties for polynomials of the third degree.

The commutativity and associativity of addition follow from the validity of these properties for the addition of numbers, since the coefficients are added at each degree of the unknown separately [2].

The commutativity of multiplication follows from the commutativity of multiplication of numbers and the fact that in the definition of the product of polynomials the coefficients of both

factors  $f(x)$  and  $g(x)$  are used equally.

The associativity of multiplication of polynomials is proved as follows: if, in addition to the

polynomials written above, a polynomial  $h(x)$  is given:

$h(x) = c_0 + c_1x + c_2x^2 + c_3x^3, c_3 \neq 0$ , then the coefficient at  $x^i, i = 1, 2, \dots, n + m + t$ , where  $t = 3$ , in the work  $[f(x)g(x)]h(x)$  will serve as a number  $\sum_{j+m=i} (\sum_{k+l=j} a_k b_l) c_m = \sum_{k+l+m=i} a_k b_l c_m$ , and in the work  $f(x)[g(x)h(x)]$  - its

equal number  $\sum_{k+j=l} a_k (\sum_{l+m=j} b_l c_m) = \sum_{k+l+m=i} a_k b_l c_m$ .

Finally, the validity of the distributivity law follows from the equality

$$\sum_{k+l=i} (a_k + b_k) c_l = \sum_{k+l=i} a_k c_l + \sum_{k+l=i} b_k c_l$$

Since the left side of the equality is the coefficient of  $x^i$  in the polynomial  $[f(x) + g(x)]h(x)$ , and the right side is the coefficient of the same degree of the unknown in the  $[f(x) + g(x)]h(x)$  [2].

Note that the role of unit in the multiplication of polynomials is played by the number 1, considered as a polynomial of degree zero.

On the other hand, a polynomial  $f(x)$  has an inverse polynomial  $f^{-1}(x)$ , if and only if

$$f(x)f^{-1}(x) = 1, \quad (3)$$

if  $f(x)$  is a polynomial of degree zero. Indeed, if  $f(x)$  is a non-zero number  $a$ , then the inverse

polynomial for it is the number  $a^{-1}$ . If  $f(x)$  has  $n \geq 1$ , then the degree of the left side of the

equality  $f(x) - g(x) = (a_0 - b_0) + (a_1 - b_1)x + (a_2 - b_2)x^2 + \dots + (a_p - b_p)x^p$

, if the polynomial  $f^{-1}(x)$  existed, would be at least  $n$ , while on the right there is a polynomial of degree zero [3, p. 133]. This implies that for the multiplication of polynomials, the inverse operation - division - is impossible.

#### List of used literature:

1. Vinberg, E. B. Algebra of polynomials: Proc. allowance for part-time students / E. B. Vinberg. - M.: Education, 1980 - 175 p.

2. Kurosh, A. G. A course in higher algebra / A. G. Kurosh. - St. Petersburg: Lan, 2003. - 432 p.
3. Solution of cubic equations. - URL: [http://www.cleverstudents.ru/equations/cubic\\_equations.html](http://www.cleverstudents.ru/equations/cubic_equations.html) (accessed 12/10/2021).