АЛГОРИТМ РЕШЕНИЯ ЗАДАЧИ КВАДРАТИЧНОГО ВЫПУКЛОГО ПРОГРАММИРОВАНИЯ МЕТОДОМ ПОСЛЕДОВАТЕЛЬНОГО ПЕРЕБОРА ГРАНЕЙ

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ALGORITHM OF DECISION OF TASK OF QUADRATIC PROTUBERANT PROGRAMMING BY METHOD OF LINEAR SEARCH OF VERGES

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????????. ???? ????????????? ?????? ??????? ??????? ? ???????????????? ???????? ?????? ???????????? ??????????? ?? 0 ?? *n*-1 ? ?????? ?? ???? ?????? ????????????????? ?????. ????? ?????????? ? ?????????, ??????? ?? ????? ??????????? ????????? ??????????? ???????, ???????.

Abstract. The essence of the proposed method lies in the sequential sorting of faces of the polyhedron of the constraints of dimension from 0 to n-1 and the faces of the quasi-stationary points. The search begins with the nearest visible point of

absolute maximum of a quadratic function, the vertex.

???????? ?????: ??????????????? ????????????????; ??????????; ?????????? ????????????????; ??????????? ??????; ????????????????? ?????.

Keywords: mathematical programming; optimization; nonlinear programming; quadratic problem; quasi-stationary point.

?????? ????????? ????????????? ???????????????? ??????????? ? ?????? ?????????? ??????????? ??????? ?? ???????? ????????? m ???????? ??????????? ?? n ??????????. ??? ??????? ????? ?????? ??????????? ????????? ??????, ?????????? ?? ??????? ????-?????? [1,2]. ?? ????? ?????? ??????? ???????? ????????????? ?????????? ? ????? ?????????? ??????? ????????????? ??????????, ??? ????????? ?????? ? ??????????? ?? ??????????. ????????? ? [3] ?????? ???????? ?????? ??????? ???????????????? ??????? ?????? ???????? ????????, ??????? ??????? ????????? $2n$. ? ?????? ?????? ????????? ?????? ????? ??????? ???????? ??????????? ?????? ? ????????????? ?????????????, ?????????? ?? ???????????????? ??????? ?????? ???????? ????????? ??????? n.

?????????? ?????? ??????????? ??????????? ???????

Z(*x*) ? ???????????? ???????????? ????????? *D* ?? ????????? ???????? ????????? ? ???????? ??????????? (??????????)

$$Z(x) = \langle D[N,N]x, x\rangle + \langle c, x\rangle \to \max_{x\in\Omega} x$$

$$\Omega = \{\, x[N] \mid A[M,N]\cdot x[N] \geq b[M] \,\}$$

.

? [4] ????????, ??? ? ??????? ???????? ? ?????????????? ??????? ?????????, ?????????? ???????? ????????????? *W*, ????? ?????? ????? ?????? ? ?????? ?????? ????? ????????? ?, ????????? ? ????? y^* ???????????? ????????? ??????? ???????, ???? $y^* \notin \Omega$

$$Z(y) = \|y - y^*\| = \langle -E[N,N]y, y\rangle + \langle p, y\rangle \to \min_{y\in\Omega} y$$

$$\Omega = \{\, y[N] \mid T[M,N]\cdot y[N] \geq b[M] \,\}$$

;

$y = W^{-1}x$; $T = AW$; $p = cW^{-1}$; $y^* = p/2$

.

???? ???????????? ?????? ??????? ??????? ? ??????????????? ???????? ?????? ?????????? ?? 0 ?? *n*-1 ? ?????? ?? ???? ?????? ???????????????? ?????.

???????????????? ????? ?? ????? ??????????? k>0 ??????????? ? ??????? ???????? ???????, ??????????? ????? y^* ? ???????????????? ?????? ?? ????? ??????????? k-1, ?? ??????? ? ????? ??????????? k-1, ????????????? ????? ??????????? k.

? ????? ?????? ????? ?????????????? ?????????? ????????? ??????? ??????????? ?? m ??????????? ??????, ??? ??????? ????????? ?? ? ?????????, ? ?????????? ???? ????????? ???????????

$$\Omega(J)=\left\{ y[N] \;\middle|\; \begin{array}{l} T[J,N]\cdot y[N]=b[J]\,,\; J\subset M, \\ T[M\setminus J,N]\cdot y[N]\geq b[M\setminus J] \end{array} \right\}$$

.

????? ??????????? 0 ??????? ?? ??????????? (????????????????) ?????, ?????????? ????????, ??? ?????????? ??????????? n ??? ????? ???????????.

$$\Omega^0(J)=\left\{ y[N] \;\middle|\; \begin{array}{l} T[J,N]\cdot y[N]=b[J]\,,\; J\subset M,\;\; |J|\geq n,\; \mathrm{rank}\;\; T[J,N]=n \\ T[M\setminus J,N]\cdot y[N]\geq b[M\setminus J] \end{array} \right\}$$

.

????? ??????????? k>0 ?????? ???????? ? ???? ????????? ????? ??????????? k-1 ? ??? ?????????? ? ??? ????? ??????? ???????????. ? ?????????, ????? ?????

??????????? 1 ?????? ???????? ??? ???????, ? ????? ????? ??????????? k>1 ?????? ???????? ?? ????? k+1 ??????. ????? ?(J_1) ??????????? k ???????? ??? ????? ?(J_2) ??????????? k+1 ??????????, ? ????? ?(J_2) ??? ????? ?(J_1) ?????????, ???? ????????? ????????? J_2 ???????? ????????????? J_1

$$J_2 \subset J_1 \subseteq M$$

.

???? ??? ??? ????? ?????? ????????? ???????? ???????? ???? ? ?? ?? ?????, ?? ????? ????? ???????? ???????????

$$\Omega(J_1) = \Omega(J_2), \quad J_1 \neq J_2, \quad |J_1| = |J_2|, \quad J_1 \subset M, \quad J_2 \subset M$$

.

????????????? ????? ??????????? k ??????????? ??????????? ????????? ??????????. ???????????, ? ???????? ??? ????? ????????????, ?????????? ???????. ? ????????????? ????? ??????????? k ??????? ?????? n-k ?????????? ??????. ???????? ?????????? ????????? J ??????? ? ??????????? ???????????? ?????, ??? $|J|$=k ??????????? ????? ????? n-k.

?????????? ?????????? ?????? ? ??????????? ????? ??????????? ????????? ?????????, ?????????? ??????????? ???? ????????? ???????? ??????????? ?????

$$r = |J_1 \cup J_2 \cup \ldots \cup J_l|, \quad \Omega(J_1) = \Omega(J_2) = \ldots = \Omega(J_l)$$

.

??? ????????? ? ???????? ?????? ??????????? n, ??? ???????? ? ???? ??? ????? ??????? ???????????, ?????????? ??????? ?, ? ????? ???????????? ? .

???? ????? ???????????? ???????? $y^* = c/2$????????? ?????? ??? ?? ??????? ? , ?? ???? $y^* \in \Omega$, ?? ??? ???????? ???????? ??????, ? ????????? ?????? ??????? ?????? ????????? ?? ??????? ? ? ??? ????? ????? ??????????????? ????????? ??????.

??????? ?????? ?????????? ? ???????, ????????? ? ????? y^*, ? ????????????, ???? ??????????????????? ????? ?? ???? ?????????? ?????? ?? ??????????????. ??????????? ?????? ?????, ??????? ?? ????? y^*, ?.?. ? ????????? ????????? ??????? ??????? ???? ?? ???? ?? ???????????, ?????????? ??? ????? y^*

$$J \cap \left\{ i \in M \;\middle|\; T[i,N] \cdot y^*[N] < b[i] \right\} \neq 0$$

.

????? ?????????? ? ?????????? ??????? ??????, ??? ??????? ???? ????????? ?????????? ?????? n ????????? ?? m ??????????? ??????

$$v(J) = \left\{ y[N] \;\middle|\; \begin{array}{l} T[J,N] \cdot y[N] = b[J],\ J \subset M,\ |J| = n, \\ T[M \setminus J,N] \cdot y[N] \geq b[M \setminus J], \\ J \cap \left\{ i \in M \;\middle|\; T[i,N] \cdot y^*[N] < b[i] \right\} \neq 0 \end{array} \right\}$$

.

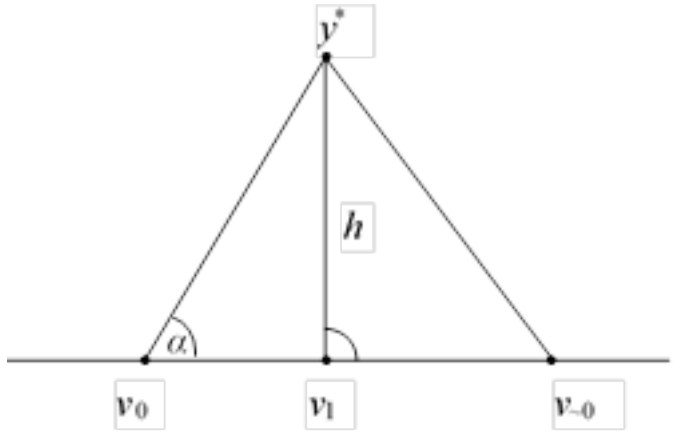
?? ???? ?????? ??????? ????????? ? y^* ? ??????? ?? v_0 , ? ?? ????????? ????????? ????????? J_0 . ???? ?????? ?????????? n ????????? ?? m ??????????? ???? ?????????? ???????, ?? ??? ?????????? ???? ???????, ??????? ??? ????????? ????????? ???? ??????? ??????? ?????????? ? ????.

????????? ??? ??????????? ? ??????? v_0 ????? ??????????? 1 (?????) ??? ???? ??????, ???? ?? ??????? v_0, ? ?????? $v_{\sim 0}$????? ? v_0 n-1 ????? ???????? ???????????

$$J_{\sim 0} \subset J_0, \quad |J_{\sim 0}| = n-1$$

.

??? ??????? ????? ????????? ?????????? ?? ???? ?? y^* ? ??????? ??????????? ?? ?????? v_0 ? $v_{\sim 0}$, ????????????? ?????, ? ????? y^*(???.1).



??????? 1. ????? ??????????????? ????? ?????

??????????? ?? ????? ???????????? ??? ?????? ????? ????????????, ????????? ?? ??????? y^* ?? ????????? $v_0 v_{\sim 0}$. ????????? ????? v_0 ????????? ? y^*, ?? ?????????????????? ????? ????? v_1 ????? ? ????? v_0 ??? ? $v_{\sim 0}$.

????????? ????????? ???????????? $y^* v_0 v_{\sim 0}$:

?????? – $h = 2S / \|v_0 - v_{\sim 0}\|$;

??????? –

$$S = \sqrt{P \cdot (P - \|v_0 - v_{\sim 0}\|) \cdot (P - \|v_0 - y^*\|) \cdot (P - \|v_{\sim 0} - y^*\|)}$$;

???????????? – $P = (\|v_0 - v_{\sim 0}\| + \|v_0 - y^*\| + \|v_{\sim 0} - y^*\|)/2$;

???? –

$$\cos\alpha = \frac{\|v_0 - v_{\sim 0}\|^2 + \|v_0 - y^*\|^2 - \|v_{\sim 0} - y^*\|^2}{2 \cdot \|v_0 - v_{\sim 0}\| \cdot \|v_0 - y^*\|}$$.

????

$$0 \le \cos\alpha \le \frac{1}{\sqrt{1 + \frac{4h^2}{\|v_0 - v_{\sim 0}\|^2}}}$$

, ?? ?? ???????????? ?????

???? ???????????????? ????? v_1, ?? ??????????

$$v_1 = v_0 + \frac{v_{\sim 0} - v_0}{\|v_{\sim 0} - v_0\|}\sqrt{\|v_0 - y^*\|^2 - h^2}$$

.

??? ???? ???? cos? = 0, ?? ???????????????? ????? ????? v_1 ????????? ? ???????? v_0 ? ??? ??????? ???????? ???????? ??????.

???? $\|v_0 - y^*\| = \|v_{\sim 0} - y^*\|$, ?? ????? v_0 ? $v_{\sim 0}$??????????? ? ???????????????? ????? ????????? ?? ???????? ????? $v_1 = (v_0 + v_{\sim 0})/2$

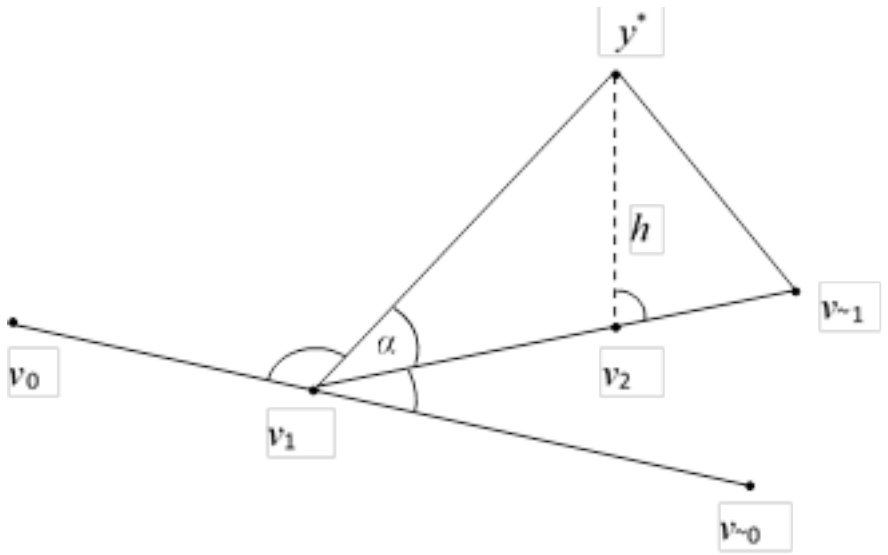
.

????? ?? ???? ?????, ??????????? ? ??????? v_0 , ??????? ????????? ? ????? y^*, ??? ????????? ????????? ????????? J_1 ? ???????? ??? ????? ??????????? 2, ?????????? ? ?????????? ?????, ?.?. ??????? ? ???? ?????? n-2 ????? ???????? ???????????. ?? ?????? ????? ????? ?????? ???????????????? ?????. ??? ????? ?? ????? v_1 ???????? ???????????? ? ?????? $v_0 v_{\sim 0}$, ??????? ?? ??????????? ?????, ? ?????? ?? ??? ??????????? ????? $v_{\sim 1}$. ??????? ????????? ??? ????? $v_{\sim 1}$????? ????????? ??????? ?????????????? ?????? $v_0 v_{\sim 0}$? $v_1 v_{\sim 1}$, ? ????? n-2 ??????? ?????????? ??????????? ??? ????? $v_{\sim 1}$. ???? ??

????????? ????? $v_{\sim 1}$??????? ?????? ??????????.

$$\langle v_0 v_{\sim 0}, v_1 v_{\sim 1} \rangle = 0;$$
$$T[J_1, N] \cdot v_{\sim 1}[N] = b[J_1], \quad J_1 \subset J_0, \quad |J_1| = n-2.$$

?? ??????? $v_1 v_{\sim 1}$???????? ??????????, ???????? ????? ??????? ? ?????? y^* (???.2). ?????? ????? ??????????? ????? ?????????? ?? ???????? ???????? ?????????? ????? ?? ????? y^* .



??????? 2. ????? ??????????????? ????? ????? ??????????? 2.

????????? ??????????? $y^* v_1 v_{\sim 1}$ – ?????? h, ??????? S, ??????????? P ? ???? cos? – ??????????? ????????? ????????? ??????????? $y^* v_0 v_{\sim 0}$.

???? cos? ? 0, ?? ????????? ?????? ???????? ????????

????? ???????? ?????

$$v_2 = v_1 + \mathrm{sign}(\cos\alpha)\frac{v_{\sim 1} - v_1}{\|v_{\sim 1} - v_1\|}\sqrt{(v_1 - y^*)^2 - h^2}$$

.

??? ???? ????? v_2 ???? ??????????????? ?????? ?????????? ???????

$$A[M \setminus J_1, N]\cdot v_2[N] \le b[M \setminus J_1], \quad J_1 \subset J_0, \quad |J_1| = n-2$$

.

???? cos? = 0 , ?? ????? v_2 ???????? ??????????????? ? ????????? ? ?????? v_1.

????? ?? ???? ?????? ??????????? 2 ??????? ?????, ??????????????? ????? ??????? ????? ???? ? ????? y^* ? ????????? ??? ?????????? ? ??? ????? ??????????? 3. ????? ??????????? ????? ??????????? 4 ? ??? ????? ?? ?????? ??????????? n-1.

??????? ?????? ????? ???????????, ???? ?? ???? ?????????? ?????? ?? ???????? ??????????????? ?????. ? ???? ?????? ???????? ?????? ????? ????????? ? y^* ?? ????? ????????? ??????????????? ?????.

? ????? ?????? ???????????? ????? ??????????? k>1, ? ????????? ?????????? J_k ?????????? ? ?????????? ??????? $v_k v_{\sim k}$? ????????? ????? ??????? ???????????, ??????? ?? ??????????? ?????. ??????? ????????? ??? ?????????? ?????

$v_{\sim k}$???????? k-1 ??????? ??????????????? ??????? ? k-1 ?????????????? ? ????? $v_{\sim k}$?????? ? n-k-1 ??????? ?????????? ??????????? ? ????? $v_{\sim k}$

$$\langle v_{i-1}v_{\sim i-1}, v_i v_{\sim i}\rangle = 0, \quad i \in \{2, 3.., k\};$$
$$T[J_k, N]\cdot v_{\sim k}[N] = b[J_k], \quad J_k \subset J_{k-1}, \quad |J_k| = n-k-1.$$

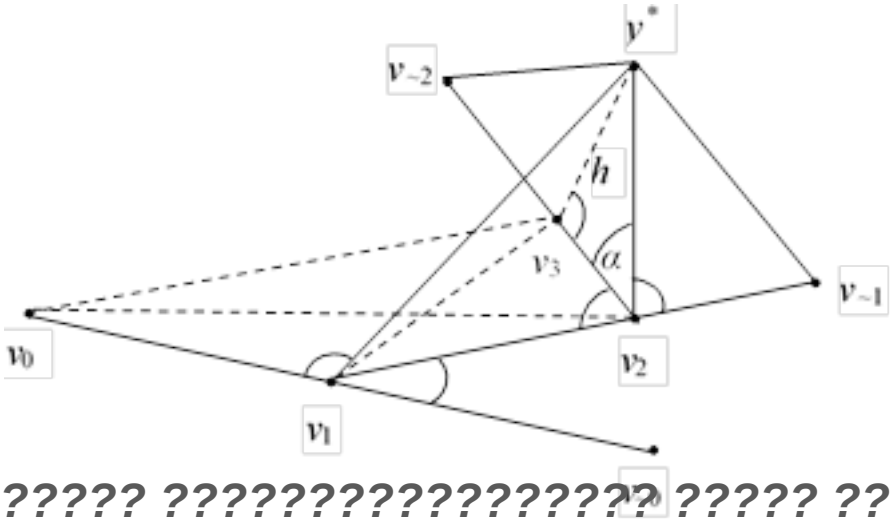
???? ?? ????????? $v_{\sim k}$??????? ?????? ??????????.

?????????????? ????? $v_{\sim k}$?????? ? ??????? v_k ? y^* ???????? ??????????, ?????? ????????, ????????? ?? ??????? ?? ????? y^* , ???????? ??????????? ?? ???????? ???????? ??????????? ?????.
????? v_{k+1} ??????? ?????? ? ?????????? ???????????, ???? c ?? ???????????, ???????? ??????????????? ??????, ???? ??? ????????????? ????????

$$T[M \setminus J_{k+1}, N]\cdot v_{k+1}[N] \le b[M \setminus J_{k+1}], \quad J_{k+1} \subset J_k, \quad |J_{k+1}| = n-k$$

.

?????????? ?????????????????? ?????????????????? ????? $v_0, v_1, .., v_{n-1}$? ??????? ?????? ?????????????? ???, ??? ?????? ??????? ??????????? ??????????? ???????? ???????, ? ?????? ??????????? ??????????? (???.3),



???.3. ????? ??????????????? ????? ?? ????? ??????????? 3

?.?. ?????????? ?? ?????? ????????? ?????????????????? ????? ????? ? y^*, ??? ?? ??????????

$$\left\|v_{i-1} - y^*\right\| \geq \left\|v_i - y^*\right\|, \quad i > 0$$

.

??????? 3 ???????? ????????? ?? 4-??????? ??????????? ?? ????????? ?????????, ??????? ?????? v_0v_3, v_1v_3, v_2,v_3 ??????????????? $v_3\,y^*$, ? ????? v_0, v_1, v_2, v_3 ?? ??????????? ????? ????????? (????? ??????????? 2).

??? ????????? ??????? ?????? ? ???????? ??????????? ? ?????????? ??????? v_i ??????? ????????? ???????? ??????????????

$$x^* = Wv_i$$

.

???????????

N – ????????? ????? ????? $\{1,2,\ldots,n\}$;

$|N|$ – ???????? ????????? N;

$\|x\| = \|x[N]\| = \sqrt{\sum_{i \in N} x[i]^2}$ – ????? ??????? x ;

$x = x[N]$ – ?????? ? ???????????? $x[i], \quad i \in N$;

$A = A[N, M]$ – ??????? ? ???????????? $A[i, j], \quad i \in N, \quad j \in M$;

$\langle x, y \rangle = x[N] \cdot y[N] = \sum_{i \in N} x[i] \cdot y[i]$ – ????????? ???????????? ???????? x ? y ;

?????? ??????????:

1. ???????? ?.?. ????????? ?????? ????????????? ????????????????. – ???: ???-?? ?????, 2004.– 127 ?.

2. ??????? ?.?., ??????? ?.?., ??????? ?.?. ???? ??????? ??????????. – ?.: ?????????, 2011. – 384 ?.

3. ????????? ?.?., ?????????? ?.?. ? ?????? ???????? ?????? ?

????????????? ?????????????????? // ??????? «DHA & CAGD». ????????? ???????.10 ???????? 2011 ?.(http://www.dha.spb.ru/).

4. ????????? ?.?. ???????? ??????? ?????? ????????????? ?????????????????? // ??????????? ???????? ? ???????.– 2014.– ?3. – ?. 36-40.