КЛАССИФИКАЦИЯ ТРЕХМЕРНЫХ АЛГЕБР СИММЕТРИИ УРАВНЕНИЯ КЛЕЙНА-ГОРДОНА В ЭЛЕКТРОМАГНИТНОМ ПОЛЕ

Белоглазов Николай Викторович

студент Омского государственного университета им. Ф.М. Достоевского (ОмГУ), РФ, г. Омск

Мячин Артем Андреевич

студент Омского государственного университета им. Ф.М. Достоевского (ОмГУ), РФ, г. Омск

Магазев Алексей Анатольевич

научный руководитель, канд. физ.-мат. наук, доц. Омского государственного технического университета (ОмГТУ), РФ, г. Омск

? ????????? ?????? ??????????????? ????????? ??????-??????? ??? ????????? ????????? ??????? ?? ??????? ???????????????? ????. ??? ??? ??? ????????? ?? ???????? ? ????? ??????, ??????????? ????????? ????????? ????????? ??? ????? ?????????. ??? ????? ???????? ?????????? ??????? ?????????, ????? ????????? 2-????? ???????????????? ????, ? ??????? ??????? ????? ????????? ????????? ?????????. ? ?????????? ?????? ?? ??????????? ?????????? ??????? ?????????? ????????? ????????? ??????-??????? ?? ??????? ???????????????? ???? ? ?????????? ?? ? ???????? ???????? — ???????? ????????? ????????? ??????-??????? ??? ????????? ?????????? ???????.

?????????? ????????? ??????-??????? ??? ????????? ?????????? ??????? ?????? $\hat{H}$.

$$\hat{H}\psi = \left(g^{ij} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} + m^2 \right) \psi = 0, \quad (1)$$

???: $\hat{H}$ — ???????? ??????????;

ψ — ???????? ??????? ???????;

g^{ij} — ??????????? ??????????? ?????? ???????????-??????? ???????????.

????????? ????????? ??????? ??????? $\hat{\xi}_\alpha$????????? (1) ????????????? ?????????? ?????????????? ??????????? [1]:

$$\left[\hat{H}, \hat{\xi}_\alpha\right] = 0. \tag{2}$$

????????, ??? ????? ????????? ????????? ???????? ??????? ?? $p(1,3)$?????? ???????? ?? ?????????? ????????????:

$$P_i = \frac{\hat{\partial}}{\partial x^i}; J_{ij} = x_i \frac{\hat{\partial}}{\partial x^j} - x_j \frac{\hat{\partial}}{\partial x^i}; \tag{3}$$

???: P_i — ?????????? ???????;

J_{ij} — ?????????? ????????.

?? ??????? ?????????????? ???? ? ?????????? $A = A_i dx^i$????????? (1) ????????? ???:

$$\hat{H}^{(\varepsilon)}\psi = \left(g^{ij}\nabla_i^{(\varepsilon)}\nabla_j^{(\varepsilon)} - m^2\right)\psi = 0, \tag{4}$$

???: $\nabla_i^{(\varepsilon)} \equiv \frac{\partial}{\partial x^i} - i\varepsilon A_i$;

ε — ????? ???????.

????? ???????????? ????????? ????????? ????????? (4) ? ????:

$$\tilde{\xi}^{(\varepsilon)}_{\alpha} = \xi^{i}_{\alpha}(x)\nabla^{(\varepsilon)}_{i} + i\varepsilon\chi_{\alpha}(x),$$ (5)

???: $\xi^{i}_{\alpha}(x), \chi_{\alpha}(x)$ — ?????????? ???????.

???????? ????????? ???????? ???? A, ?????? ??? ???????

$\xi^{i}_{\alpha}(x), \chi_{\alpha}(x)$.

????????? ????????? (5) ???????????? ?????????? ?????????????? ??????????:

$$\left[\hat{H}, \tilde{\xi}^{(\varepsilon)}_{\alpha}\right] = 0.$$ (6)

??????? (6) ?????????? ??????? ?????????:

$$\begin{cases} L_{\xi_\alpha} g = 0, \\ d\chi_{\alpha} + i_{\xi_\alpha} F = 0, \end{cases}$$ (7)

???: $F = dA$ — ????????? 2-????? ???????? ???????????????? ????;

L_{ξ_α} — ?????????? ?? ????? ????????? ???? ξ_{α};

i_{ξ_α} — ???????? ?????????? ???????????.

????????? (7) ????? ????????? ? ?????? [2], ?? ????????? ??????? ????????? ????????? ? ???? ?????? ?? ?????????.

??????? ??????? ????????? ??????? (7) — ??? ??????? ???????? ???? (3). ??????? ?? ??? ?? ???????, ??????? ????????? 2-????? F ???????? ???????????????? ????, ?? ???? ?????????????? ???????:

$$L_{\xi_\alpha} F = 0.$$ (8)

????????? ????? ???????? ???????? ????????? ?????????? g ? ??????? $p(1,3)$.

????? ?????? ????????? ??????? (7) ? ?????????? ????????? ξ_α, ???????:

$$\chi_\alpha = -\int i_{\xi_\alpha} F. \tag{9}$$

???????? ? (9) ????????? ? ???? ??????? (8).

????????? ????????? ??????? $\xi_\alpha(x),\ \chi_\alpha(x),$?????? ????????? ????????? ??????? ??????? ????????? ??????-??????? ???????? (5). ?????? ????????? ???????? ????????? ???????g, ? ????? ?????? ???????? ?? ??????? ???????? $p(1,3)$.

?????? ????????? ??????? g. ?????????? $\xi_\alpha^{(\varepsilon)}$? $\xi_\beta^{(\varepsilon)}$????? ???????? ???

$$\left[\xi_\alpha^{(\varepsilon)}, \xi_\beta^{(\varepsilon)}\right] = C_{\alpha\beta}^{\gamma}\,\xi_\gamma^{(\varepsilon)} + i\varepsilon\,\Omega_{\alpha\beta}, \tag{10}$$

???: $C_{\alpha\beta}^{\gamma}$ — ?????????? ????????? ??????? g;

$\Omega_{\alpha\beta}$????? ???:

$$\Omega_{\alpha\beta} = F(\xi_\alpha, \xi_\beta) - C_{\alpha\beta}^{\gamma}\chi_\gamma. \tag{11}$$

????? ????????, ??? $\Omega_{\alpha\beta}$???????? 2-?????? ??????? ?? g [3]. ?????,

??? ? ????? ?????? ??????? $\overline{g}$??????????? ??????? g.

???? ???????? ?????????? λ_{γ} ?????, ???

$$\Omega_{\alpha\beta} = C^{\gamma}_{\alpha\beta}\lambda_{\gamma},$$ (12)

?????

$$\left[\hat{\xi}^{(\varepsilon)}_{\alpha}, \hat{\xi}^{(\varepsilon)}_{\beta}\right] = C^{\gamma}_{\alpha\beta}\left(\hat{\xi}^{(\varepsilon)}_{\gamma} + i\varepsilon\lambda_{\gamma}\right).$$ (13)

????? ??????? ????? ????????? ?????????

$$\hat{\eta}^{(\varepsilon)}_{\alpha} = \hat{\xi}^{(\varepsilon)}_{\alpha} + i\varepsilon\lambda_{\alpha}$$ (14)

???????? ??????? ?? ?????????? ??????? g:

$$\left[\hat{\eta}^{(\varepsilon)}_{\alpha}, \hat{\eta}^{(\varepsilon)}_{\beta}\right] = C^{\gamma}_{\alpha\beta}\hat{\eta}^{(\varepsilon)}_{\gamma}, \quad \hat{\eta}^{(\varepsilon)}_{\alpha} \in \overline{g}.$$ (15)

?????? $\Omega_{\alpha\beta}$???? (12) ????????? ???????????.

?? (15) ???????? ????????, ??? ? ?????? ???????????? ??????? $\Omega_{\alpha\beta}$ «?????????» ???????? ???? ?? ?????? ?? ????????? ??????? ?????????? ?????????.

??? ??????????? ???????????? ???? ????????? ?????????? ?????????? ?????????? g ??????? ???????? $p(1,3)$?? ?????????? ?????????????? ?????????????:

$$\left[\xi_1, \xi_2\right] = -\xi_3, \left[\xi_1, \xi_3\right] = -\xi_2,$$ (16)

????????? ????????? (4) ? ??????????? ???????? ???????????????? ????

$$A = f(x,y)dy + \alpha t dz, \tag{17}$$

???: $f(x,y)$ — ??????????? ???????;

α — ??????????? ??????????.

??????? ?????????? ???????????? 2-?????

$$F = f_x(x,y)dx \wedge dy + \alpha dt \wedge dz.$$

????? ?????? ????????? ??????? (7) ????????? ? ???????? (8), ???????? ??????? ????????:

$$\xi_1 = z\frac{\partial}{\partial t} + t\frac{\partial}{\partial z}, \xi_2 = \frac{\partial}{\partial t}, \xi_3 = \frac{\partial}{\partial z}. \tag{19}$$

??? ?????, ??????? ??????? ??????? $\xi_\alpha(x)$.

????????? ????????? ??????? ????????, ??????? ????????? ???????

$\chi_\alpha(x)$:

$$\chi_1 = -\int(-\alpha t dt + \alpha z dz) = \frac{\alpha(t^2 - z^2)}{2}, \tag{20}$$

$$\chi_2 = -\int \alpha dz = -\alpha z, \tag{21}$$

$$\chi_3 = -\int(-\alpha dt) = \alpha t. \tag{22}$$

?????? ?? ????? ????????? ????????? ????????? ????????? ??????-??????? ?? ??????? ???????????????? ????:

$$\xi_1^{(\varepsilon)} = z\frac{\partial}{\partial t} + t\frac{\partial}{\partial z} - \frac{i\varepsilon\alpha(t^2 + z^2)}{2}, \tag{23}$$

$$\hat{\xi}_2^{(e)} = \frac{\partial}{\partial t} - i\varepsilon\alpha z,$$ (24)

$$\hat{\xi}_3^{(e)} = \frac{\partial}{\partial z},$$ (25)

??????? ????????? ??????????? ?????????? ????????? (23) — (25), ???????? ??????????? ????????? (17) ??????? g:

$$\left[\hat{\xi}_1^{(e)}, \hat{\xi}_2^{(e)}\right] = -\hat{\xi}_3^{(e)},$$ 26)

$$\left[\hat{\xi}_1^{(e)}, \hat{\xi}_3^{(e)}\right] = -\hat{\xi}_2^{(e)},$$ (27)

$$\left[\hat{\xi}_2^{(e)}, \hat{\xi}_3^{(e)}\right] = i\varepsilon\alpha.$$ (28)

?? (26) — (28) ??????? ??????, ???????? ???????? ????????????????? $\Omega_{\alpha\beta} = -\Omega_{\beta\alpha}$:

$$\Omega_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha \\ 0 & -\alpha & 0 \end{pmatrix}.$$ (29)

????????? $C_{23}^1 = C_{23}^2 = C_{23}^3 = 0,$?? ?????????? ????? ????? ????????

$\lambda = (\lambda_1, \lambda_2, \lambda_3)$, ???

$$\Omega_{23} = C_{23}^1\lambda_1 + C_{23}^2\lambda_2 + C_{23}^3\lambda_3 = \alpha,$$ (30)

?, ?????????????, 2-?????? (29) ???????? ?????????????. ????????? (23) —

(25) ???????? ??????? ????????? ???????????? ???????? ?????????? g

????????? ??????-??????? (1).

? ????????? ?????? ???? ????????? ????????? ????????? ????????? ??????-??????? ??? ?????????? ????????? ??????? ? ???????????????? ????. ??????????, ??? ??????? ?????????? ????????? ??????? ????????? ? ????? ?????? ??????????? ??????? ????????, ???????????? ??????????? ????????? ????????? ??????-??????? ??? ????????? ???????. ?????????? ????????? ????????? ??? «?????????» ???????? ???? ???????????? ????????????? ???????????????? 2-???????. ??? ???? ?????????? ????????? ??????? ????????, ? ???????????? ? ?????????? ? ????????? ?????? ??????????, ???? ????????? ????????? ????????? ??????? ???????, ???????? ?????? ?? ?????????? ??? ????? ?? ?????????.

?????? ??????????:

1. ????? ?.?., ??????? ?.?. ????????? ????????? ????????? ????????. — 1990.
2. Van Holten J.W. Covariant hamiltonian dynamics // Physical Review D. — 2007. — ?. 75. — ? 2. — ?. 025027.
3. ???? ?., ????????? ?. ??????????? ??????? ??. — 1981.